انتقال حرارت ۲

جلسه سوم

مدرس:
"مریم براتی"
"دکتری مهندسی شیمی"
"دانشکده فنی و حرفه ای شهید رجایی"

ادامه فصل اول:

جابجایی آزاد

استوانه افقی طویل(Long Horizontal Cylinder)

رابطه مورگان(Morgan) در مورد این شکل هندسی بصورت زیر است:

$$N\bar{u}_D = \frac{\bar{h}D}{k} = CRa_D^n$$

که در آن C و n از جدول مخصوص بدست می آیند و عدد رایلی و نوسلت بر مبنای قطر استوانه اند.

$$Ra_D = Gr_D \Pr = \frac{g\beta(T_s - T_\infty)D^3}{\nu\alpha}$$

چرچیل و چو رابطه ای بصورت زیر پیشنهاد داده اند:

$$N\bar{u} = \left[0.6 + \frac{0.387Ra^{\frac{1}{6}}}{\left[1+\left(\frac{0.559}{\Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{9}}}\right]^2 \qquad 10^{-5} < Gr\Pr < 10^{-12}$$

مثال:

یک هیتر افقی به قطر ۲ سانتی متر و دمای سطح 38^0C در آبی با دمای 27^0C غوطه ور شده است

میزان انتقال حرارت جابجایی آزاد بر واحد طول هیتر را محاسبه کنید.

1

$$T_f = \frac{(T_w + T_\infty)}{2} \rightarrow T_f = \frac{(38+27)}{2} = 32.5^0C$$

2

$$k = 0.623 \qquad \frac{g\beta\rho^2 c_p}{\mu k} = 2.48\times10^{10}$$

3

$$Gr.\Pr = \frac{g\beta\rho^2 c_p}{\mu k}(T_w - T_\infty)D^3$$

$$Gr.\Pr = 2.48\times10^{10}(38-27)(0.02)^3 = 2.18\times10^6$$

Table A-9 Properties of Water (Saturated Liquid)†

Note: $\mathrm{Gr}_x\mathrm{Pr} = \left(\frac{g\beta\rho^2 c_p}{\mu k}\right)x^3\Delta T$

°F	°C	c_p, kJ/kg · °C	ρ, kg/m³	μ, kg/m · s	k, W/m · °C	Pr	$\frac{g\beta\rho^2 c_p}{\mu k}$, 1/m³ · °C
32	0	4.225	999.8	1.79×10^{-3}	0.566	13.25	
40	4.44	4.208	999.8	1.55	0.575	11.35	1.91×10^{9}
50	10	4.195	999.2	1.31	0.585	9.40	6.34×10^{9}
60	15.56	4.186	998.6	1.12	0.595	7.88	1.08×10^{10}
70	21.11	4.179	997.4	9.8×10^{-4}	0.604	6.78	1.46×10^{10}
80	26.67	4.179	995.8	8.6	0.614	5.85	1.91×10^{10}
90	32.22	4.174	994.9	7.65	0.623	5.12	2.48×10^{10}
100	37.78	4.174	993.0	6.82	0.630	4.53	3.3×10^{10}
110	43.33	4.174	990.6	6.16	0.637	4.04	4.19×10^{10}
120	48.89	4.174	988.8	5.62	0.644	3.64	4.89×10^{10}
130	54.44	4.179	985.7	5.13	0.649	3.30	5.66×10^{10}
140	60	4.179	983.3	4.71	0.654	3.01	6.48×10^{10}
150	65.55	4.183	980.3	4.3	0.659	2.73	7.62×10^{10}
160	71.11	4.186	977.3	4.01	0.665	2.53	8.84×10^{10}
170	76.67	4.191	973.7	3.72	0.668	2.33	9.85×10^{10}
180	82.22	4.195	970.2	3.47	0.673	2.16	1.09×10^{11}

Table 7-1 Constants for Use with Eq. (7-25) for Isothermal Surfaces.

Geometry	$Gr_f Pr_f$	*C*	*m*	*Ref(s).*
Vertical planes and cylinders	10^{-1}–10^{4}	Use Fig. 7-7	Use Fig. 7-7	4
	10^{4}–10^{9}	0.59	$\frac{1}{4}$	4
	10^{9}–10^{13}	0.021	$\frac{2}{5}$	30
	10^{9}–10^{13}	0.10	$\frac{1}{3}$	22, 16†
Horizontal cylinders	0–10^{-5}	0.4	0	4
	10^{-5}–10^{4}	Use Fig. 7-8	Use Fig. 7-8	4
	10^{4}–10^{9}	0.53	$\frac{1}{4}$	4
	10^{9}–10^{12}	0.13	$\frac{1}{3}$	4
	10^{-10}–10^{-2}	0.675	0.058	76†
	10^{-2}–10^{2}	1.02	0.148	76†
	10^{2}–10^{4}	0.850	0.188	76
	10^{4}–10^{7}	0.480	$\frac{1}{4}$	76
	10^{7}–10^{12}	0.125	$\frac{1}{3}$	76
Upper surface of heated plates or lower surface of cooled plates	2×10^{4}–8×10^{6}	0.54	$\frac{1}{4}$	44, 52
Upper surface of heated plates or lower surface of cooled plates	8×10^{6}–10^{11}	0.15	$\frac{1}{3}$	44, 52
Lower surface of heated plates or upper surface of cooled plates	10^{5}–10^{11}	0.27	$\frac{1}{4}$	44, 37, 75
Vertical cylinder, height = diameter Characteristic length = diameter	10^{4}–10^{6}	0.775	0.21	77
Irregular solids, characteristic length = distance fluid particle travels in boundary layer	10^{4}–10^{9}	0.52	$\frac{1}{4}$	78

4

$$Nu = 0.53(2.18\times10^{6})^{\frac{1}{4}} = 38.425$$

5

$$Nu_{xf} = \frac{hD}{k} = 38.425$$

$$\frac{h(0.02)}{0.623} = 38.425 \rightarrow h = \frac{38.425\times0.623}{0.02} = 1196.94$$

6

$$q = hA(T_w - T_\infty) = h(\pi DL)(T_w - T_\infty)$$

$$q = 1196.94(3.14\times0.02\times1)(38-27) = 826.84$$

مثال:

یک سیم حاوی جریان الکتریکی به قطر ۰/۰۲ میلیمتر با دمای ثابت 54^0C در معرض هوایی با دمای 0^0C قرار دارد میزان توان الکتریکی لازم که این سیم در این دما قرار گیرد در شرایطی که طول سیم ۵۰ متر باشد را محاسبه کنید.

1. $T_f = \frac{(T_w + T_\infty)}{2} = \frac{(54+0)}{2} = 27^0C \rightarrow T_f = 273+27 = 300K$

2. $\upsilon = 15.69\times10^{-6}\ \ k = 0.02624 \quad \Pr = 0.708$

3. $\beta = \frac{1}{T_f} = \frac{1}{300} = 3.333\times10^{-3}$

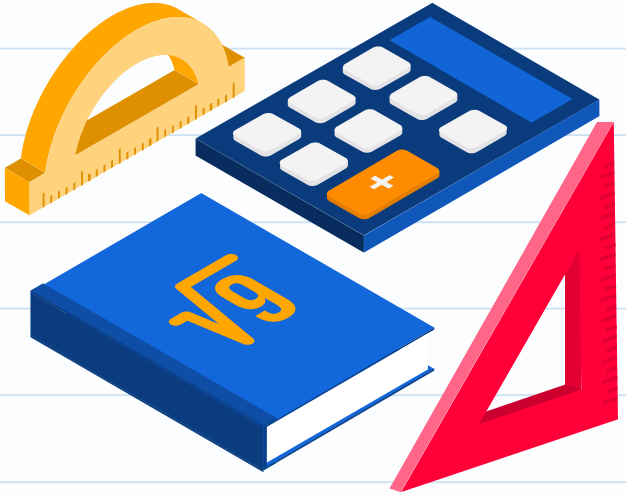

Table A-5 Properties of Air at Atmospheric Pressure†

The values of μ, k, c_p, and Pr are not strongly pressure-dependent and may be used over a fairly wide range of pressures.

T, K	ρ kg/m³	c_p, kJ/kg · °C	μ, kg/m · s × 10^5	ν, m²/s × 10^6	k, W/m · °C	α, m²/s × 10^4	Pr
100	3.6010	1.0266	0.6924	1.923	0.009246	0.02501	0.770
150	2.3675	1.0099	1.0283	4.343	0.013735	0.05745	0.753
200	1.7684	1.0061	1.3289	7.490	0.01809	0.10165	0.739
250	1.4128	1.0053	1.5990	11.31	0.02227	0.15675	0.722
300	1.1774	1.0057	1.8462	15.69	0.02624	0.22160	0.708
350	0.9980	1.0090	2.075	20.76	0.03003	0.2983	0.697
400	0.8826	1.0140	2.286	25.90	0.03365	0.3760	0.689
450	0.7833	1.0207	2.484	31.71	0.03707	0.4222	0.683
500	0.7048	1.0295	2.671	37.90	0.04038	0.5564	0.680
550	0.6423	1.0392	2.848	44.34	0.04360	0.6532	0.680
600	0.5879	1.0551	3.018	51.34	0.04659	0.7512	0.680
650	0.5430	1.0635	3.177	58.51	0.04953	0.8578	0.682
700	0.5030	1.0752	3.332	66.25	0.05230	0.9672	0.684
750	0.4709	1.0856	3.481	73.91	0.05509	1.0774	0.686
800	0.4405	1.0978	3.625	82.29	0.05779	1.1951	0.689
850	0.4149	1.1095	3.765	90.75	0.06028	1.3097	0.692
900	0.3925	1.1212	3.899	99.3	0.06279	1.4271	0.696
950	0.3716	1.1321	4.023	108.2	0.06525	1.5510	0.699
1000	0.3524	1.1417	4.152	117.8	0.06752	1.6779	0.702
1100	0.3204	1.160	4.44	138.6	0.0732	1.969	0.704

4

$$Gr = \frac{g\beta(T_w - T_\infty)D^3}{\upsilon^2} \rightarrow Gr = \frac{(9.8)(3.333\times10^{-3})(54-0)(0.02\times10^{-3})^3}{(15.69\times10^{-6})^2} = 5.73\times10^{-5}$$

5

$$Gr.\Pr = (5.73\times10^{-5})\times0.708 = 4.06\times10^{-5}$$

6

$$Nu = 0.675(4.06\times10^{-5})^{0.058} = 0.375$$

7

$$Nu_{xf} = \frac{hD}{k} = 0.375$$

$$\frac{h(0.02\times10^{-3})}{0.02624} = 0.375 \rightarrow h = \frac{0.375\times0.02624}{0.02\times10^{-3}} = 492.6$$

8

$$q = hA(T_w - T_\infty) = h(\pi DL)(T_w - T_\infty)$$

$$q = 492.6(3.14\times0.02\times10^{-3}\times0.5)(54-0) = 0.836$$

Table 7-1 Constants for Use with Eq. (7-25) for Isothermal Surfaces.

Geometry	$Gr_f Pr_f$	C	m	*Ref(s).*
Vertical planes and cylinders	10^{-1}–10^{4}	Use Fig. 7-7	Use Fig. 7-7	4
	10^{4}–10^{9}	0.59	$\frac{1}{4}$	4
	10^{9}–10^{13}	0.021	$\frac{2}{5}$	30
	10^{9}–10^{13}	0.10	$\frac{1}{3}$	22, 16†
Horizontal cylinders	0–10^{-5}	0.4	0	4
	10^{-5}–10^{4}	Use Fig. 7-8	Use Fig. 7-8	4
	10^{4}–10^{9}	0.53	$\frac{1}{4}$	4
	10^{9}–10^{12}	0.13	$\frac{1}{3}$	4
	10^{-10}–10^{-2}	0.675	0.058	76†
	10^{-2}–10^{2}	1.02	0.148	76†
	10^{2}–10^{4}	0.850	0.188	76
	10^{4}–10^{7}	0.480	$\frac{1}{4}$	76
	10^{7}–10^{12}	0.125	$\frac{1}{3}$	76
Upper surface of heated plates or lower surface of cooled plates	2×10^{4}–8×10^{6}	0.54	$\frac{1}{4}$	44, 52
Upper surface of heated plates or lower surface of cooled plates	8×10^{6}–10^{11}	0.15	$\frac{1}{3}$	44, 52
Lower surface of heated plates or upper surface of cooled plates	10^{5}–10^{11}	0.27	$\frac{1}{4}$	44, 37, 75
Vertical cylinder, height = diameter Characteristic length = diameter	10^{4}–10^{6}	0.775	0.21	77
Irregular solids, characteristic length = distance fluid particle travels in boundary layer	10^{4}–10^{9}	0.52	$\frac{1}{4}$	78

مثال:

یک میله افقی با قطر ۰/۳۰۴۸ متر و با دمای 250^0C در اتاقی قرار دارد که دمای هوای اتاق 15^0C قرار دارد میزان حرارت از دست داده شده از طریق جابجایی آزاد توسط این لوله را در شرایطی که طول میله یک متر باشد بدست آورید.

1. $$T_f = \frac{(T_w + T_\infty)}{2} = \frac{(250+15)}{2} = 132.5^0C \rightarrow T_f = 273+132.5 = 405.5K$$

2. $$\upsilon = 25.9\times10^{-6} \quad k = 0.03365 \quad \Pr = 0.689$$

3. $$\beta = \frac{1}{T_f} = \frac{1}{405.5} = 2.466\times10^{-3}$$

Table A-5 Properties of Air at Atmospheric Pressure†

The values of μ, k, c_p, and Pr are not strongly pressure-dependent and may be used over a fairly wide range of pressures.

T, K	ρ kg/m³	c_p, kJ/kg · °C	μ, kg/m · s × 10^5	ν, m²/s × 10^6	k, W/m · °C	α, m²/s × 10^4	Pr
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150	2.3675	1.0099	1.0283	4.343	0.013735	0.05745	0.753
200	1.7684	1.0061	1.3289	7.490	0.01809	0.10165	0.739
250	1.4128	1.0053	1.5990	11.31	0.02227	0.15675	0.722
300	1.1774	1.0057	1.8462	15.69	0.02624	0.22160	0.708
350	0.9980	1.0090	2.075	20.76	0.03003	0.2983	0.697
400	0.8826	1.0140	2.286	25.90	0.03365	0.3760	0.689
450	0.7833	1.0207	2.484	31.71	0.03707	0.4222	0.683
500	0.7048	1.0295	2.671	37.90	0.04038	0.5564	0.680
550	0.6423	1.0392	2.848	44.34	0.04360	0.6532	0.680
600	0.5879	1.0551	3.018	51.34	0.04659	0.7512	0.680
650	0.5430	1.0635	3.177	58.51	0.04953	0.8578	0.682
700	0.5030	1.0752	3.332	66.25	0.05230	0.9672	0.684
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1000	0.3524	1.1417	4.152	117.8	0.06752	1.6779	0.702
1100	0.3204	1.160	4.44	138.6	0.0732	1.969	0.704

4

$$Gr = \frac{g\beta(T_w - T_\infty)D^3}{\upsilon^2} \rightarrow Gr = \frac{(9.8)(2.466\times10^{-3})(250-15)(0.3048)^3}{(25.9\times10^{-6})^2} = 2.4\times10^8$$

5

$$Gr.\Pr = (2.4\times10^8)\times0.689 = 1.65\times10^8$$

6

$$Nu = 0.52(1.65\times10^8)^{\frac{1}{4}} = 58.93$$

7

$$Nu_{xf} = \frac{hD}{k} = 58.93$$

$$\frac{h(0.3048)}{0.03365} = 58.93 \rightarrow h = \frac{58.93\times0.03365}{0.3048} = 6.5$$

8

$$q = hA(T_w - T_\infty) = h(\pi DL)(T_w - T_\infty)$$

$$q = 6.5(3.14\times0.3048\times1)(250-15) = 1461.92$$

Table 7-1 Constants for Use with Eq. (7-25) for Isothermal Surfaces.

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Vertical planes and cylinders	10^{-1}–10^{4}	Use Fig. 7-7	Use Fig. 7-7	4
	10^{4}–10^{9}	0.59	$\frac{1}{4}$	4
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	10^{9}–10^{12}	0.13	$\frac{1}{3}$	4
	10^{-10}–10^{-2}	0.675	0.058	76†
	10^{-2}–10^{2}	1.02	0.148	76†
	10^{2}–10^{4}	0.850	0.188	76
	10^{4}–10^{7}	0.480	$\frac{1}{4}$	76
	10^{7}–10^{12}	0.125	$\frac{1}{3}$	76
Upper surface of heated plates or lower surface of cooled plates	2×10^{4}–8×10^{6}	0.54	$\frac{1}{4}$	44, 52
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